

# Radiative Processes in High-Energy Astrophysics

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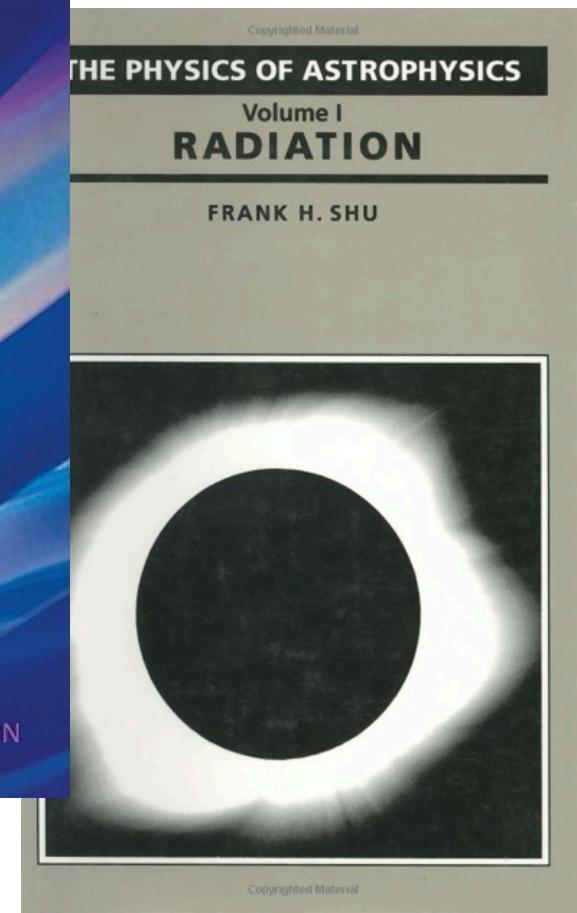
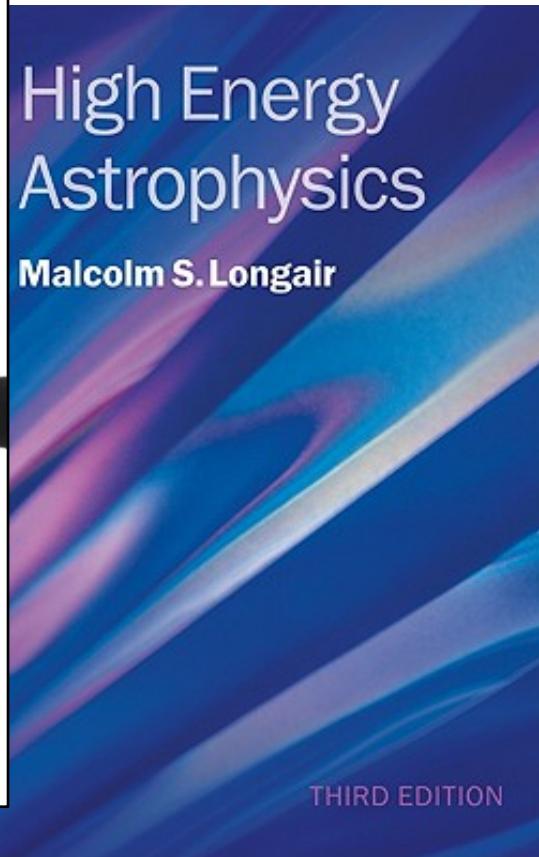
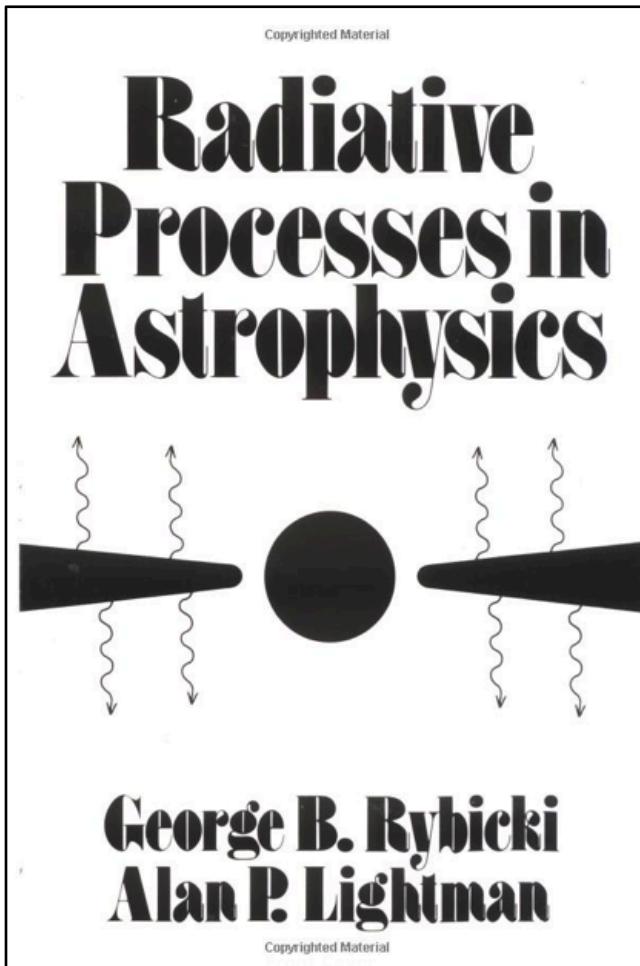
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# Outline

A very basic introduction to the selected radiative processes which are of the main importance in high-energy astrophysics

- Basic definitions and concepts
- Synchrotron Emission
- Inverse-Compton Emission
- Thermal Bremsstrahlung
- Proton-Proton Interactions
- Photo-Meson Production
- Photon-Photon Annihilation

# I. Basic Definitions and Concepts



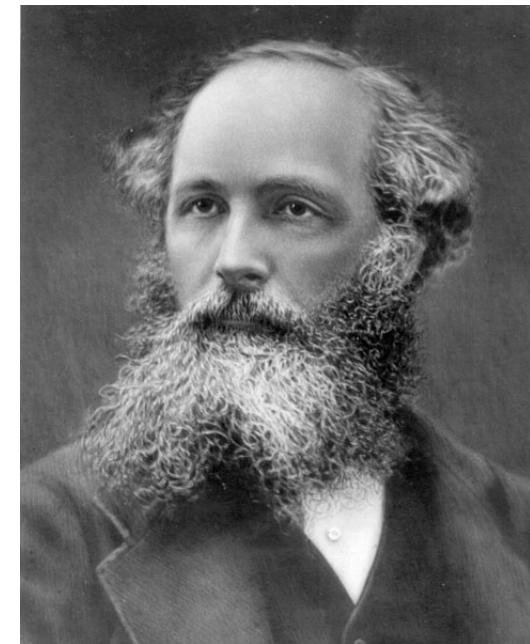
# I. Maxwell Equations

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial}{\partial t} \vec{E} \quad (1)$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \vec{B} \quad (2)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (3)$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (4)$$



$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \vec{E} = 0 \quad \text{and} \quad \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \vec{B} = 0 \quad (5)$$

$$\vec{E} = \sum_{\pm} \vec{E}_{\pm}(\zeta \pm ct) \quad \text{and} \quad \vec{B} = \sum_{\pm} \vec{B}_{\pm}(\zeta \pm ct) \quad (6)$$

Propagation of EM signal in vacuum is described by the Maxwell equations (1-4). EF and MF satisfy the homogeneous wave equation (5); general solutions (6) consist of plane waves propagating at the speed of light  $c$ .

# I. Electromagnetic Waves

Fourier decomposition

$$\vec{E}(\vec{r}, t) = \frac{1}{(2\pi)^3} \int d^3k \hat{e}(\vec{k}) \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \quad (7)$$

$$\vec{B}(\vec{r}, t) = \frac{1}{(2\pi)^3} \int d^3k \underbrace{\hat{b}(\vec{k})}_{\text{monochromatic plane wave}} \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \quad (8)$$

$$v_{\text{ph}} \equiv \omega/k = c$$

$$v_g \equiv \partial\omega/\partial k = c$$

$$\hat{k} \times \hat{e} = \hat{b}$$

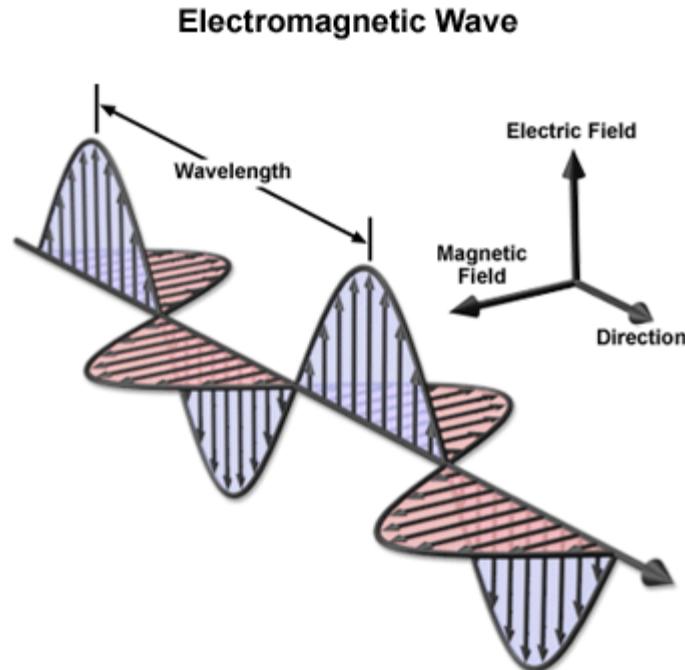
$$\hat{k} \times \hat{b} = -\hat{e}$$

$$\hat{k} \cdot \hat{e} = 0$$

$$\hat{k} \cdot \hat{b} = 0$$

$$\vec{F}_M \propto \vec{E} \times \vec{B}$$

where  $\hat{k} \equiv \vec{k}/k$



Waves with frequency  $\omega$  travelling in the direction given by the wave vector  $k$  with the phase and group velocities equal  $c$ ; electric and magnetic vectors are both transverse to the direction of wave propagation, perpendicular to each other, and equal in magnitude; the associated Poynting flux points along  $k$ .

# I. Basic Definitions

monochromatic specific intensity: the amount of radiant energy  $dE$  which crosses in time  $dt$  the area  $dA$  normal to a given direction, within a solid angle  $d\Omega$

$$I_\nu = \frac{dE}{dA d\Omega d\nu dt} \quad (9)$$

monochromatic energy flux

$$S_\nu = \oint I_\nu d\Omega \quad (10)$$

monochromatic emission coefficient (emissivity): the energy  $dE$  emitted per unit time  $dt$  per unit solid angle  $d\Omega$  per unit volume of the emitting matter  $dV$

$$j_\nu = \frac{dE}{dV d\Omega d\nu dt} \quad (11)$$

optical depth  $\tau$  of the medium through which the radiation is propagating, absorption coefficient  $\alpha$  and cross section  $\sigma$

$$\tau_\nu(L) = \int_0^L n_{ab}(\ell) \sigma_\nu d\ell \equiv \int_0^L \alpha_\nu(\ell) d\ell \quad (12)$$

# I. Radiative Transfer

$$\left( \frac{d}{d\tau_\nu} + 1 \right) I_\nu = \frac{j_\nu}{\alpha_\nu} \quad (13)$$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} \frac{j_\nu}{\alpha_\nu} d\tau'_\nu \quad (14)$$

Radiative transfer theory describes electromagnetic radiation propagating along straight lines from the source to the observer

$$I_\nu(0) \neq 0, j_\nu = 0, \alpha_\nu = 0 \rightarrow I_\nu(L) = I_\nu(0) \quad (15)$$

$$I_\nu(0) \neq 0, j_\nu = 0, \alpha_\nu \neq 0 \rightarrow I_\nu(L) = I_\nu(0) e^{-\tau_\nu(L)} \quad (16)$$

$$I_\nu(0) = 0, j_\nu \neq 0, \alpha_\nu = 0 \rightarrow I_\nu(L) = \int_0^L j_\nu(\ell) d\ell \quad (17)$$

$$I_\nu(0) = 0, j_\nu = const, \alpha_\nu = const \rightarrow I_\nu(L) = \frac{j_\nu}{\alpha_\nu} [1 - e^{-\alpha_\nu L}] \quad (18)$$

# I. Relativistic Beaming

viewing angle  $\vec{\beta} \cdot \hat{k} = \beta \cos \theta$  (19)

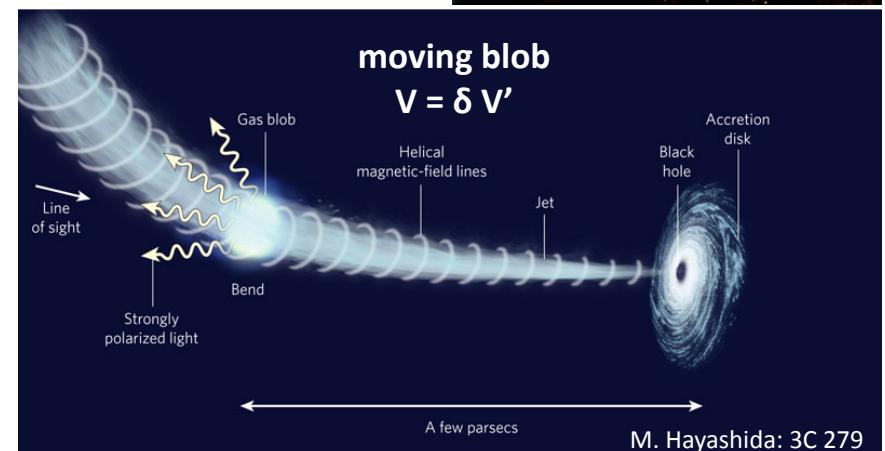
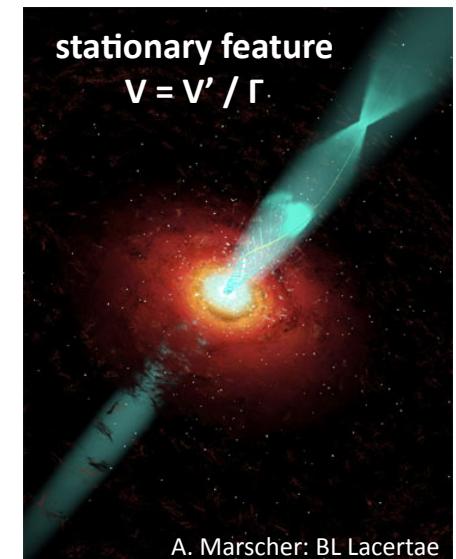
Doppler factor  $\delta \equiv \frac{1}{\Gamma(1 - \beta \cos \theta)}$  (20)

Lorentz factor  $\Gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$  (21)

$$\nu = \delta \nu' \quad (22)$$

$$\frac{I_\nu}{\nu^3} = \text{inv} \rightarrow I_\nu = \delta^3 I'_{\nu'} \quad (23)$$

$$\begin{aligned} j_\nu &= \delta^2 j'_{\nu'} \\ \alpha_\nu &= \delta^{-1} \alpha'_{\nu'} \\ d\Omega &= \delta^{-2} d\Omega' \\ \cos \theta &= (\cos \theta' + \beta) / (1 + \beta \cos \theta') \\ dV &= dV'/\Gamma \quad \text{or} \quad dV = \delta dV' \end{aligned}$$



# I. Transformations

$$\delta \rightarrow \delta/(1+z)$$
$$I \equiv \int I_\nu d\nu, \text{ etc.}$$

$$I = \left( \frac{\delta}{1+z} \right)^4 I' \quad \text{and} \quad I' = \int j' ds' \quad (24)$$

$$d\Omega = dA d_\theta^{-2} = dA (1+z)^4 d_L^{-2} \quad (25)$$

$$dV' = dA' ds' = dA ds' \quad (26)$$

$$S = \int I d\Omega = \delta^4 d_L^{-2} \int I' dA = \delta^4 d_L^{-2} \int j' dV' \quad (27)$$

$$L' \equiv \oint \frac{\partial L'}{\partial \Omega'} d\Omega' \quad \text{with} \quad \frac{\partial L'}{\partial \Omega'} \equiv \frac{L'}{4\pi} \equiv \int j' dV' \quad (28)$$

$$L_{\text{iso}} \equiv 4\pi d_L^2 S = \delta^4 L' \quad (29)$$

$$L_{\text{em}} \equiv \oint \frac{L_{\text{iso}}}{4\pi} d\Omega \simeq \Gamma^2 L' \quad (30)$$

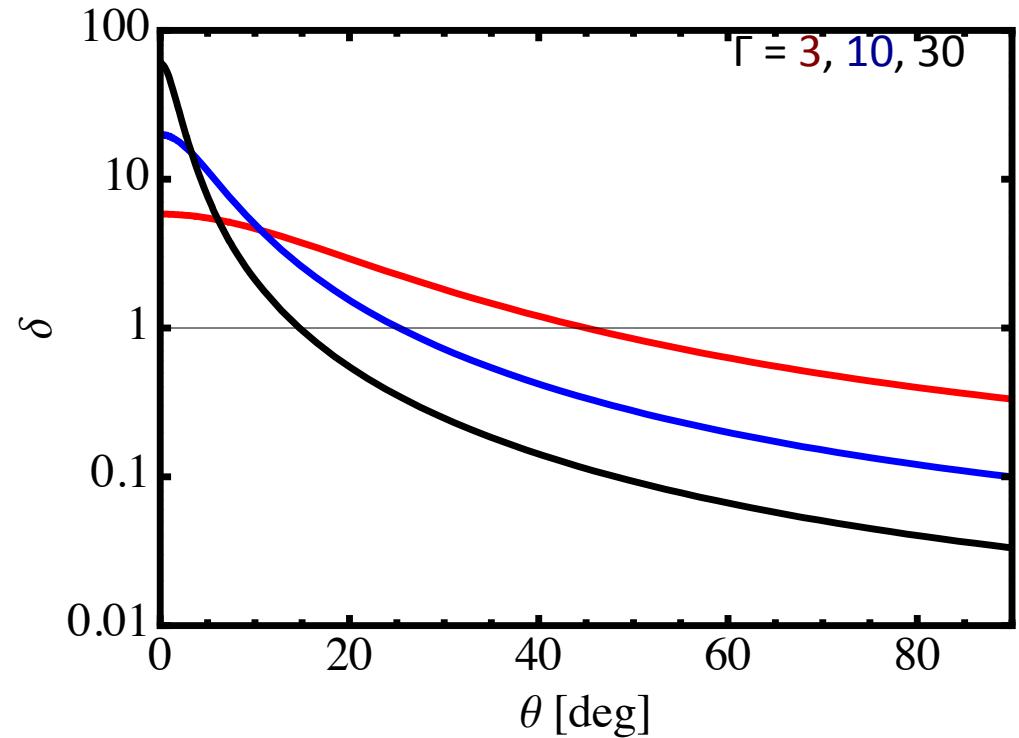
# I. Summary

$$\delta = 1 / \Gamma (1 - \beta \cos\theta)$$

$$\delta \approx \Gamma \text{ for } \theta \leq 1/\Gamma$$

Assuming that the emission is isotropic in the source rest frame:

- Observed frequency  $v = \delta v' / (1+z)$
- Intrinsic luminosity  $L' = 4\pi j' V'$
- Observed energy flux  $S = \delta^4 L' / 4\pi d_L^2$
- “Isotropic” luminosity  $L_{\text{iso}} = \delta^4 L'$
- Total emitted power  $L_{\text{em}} = \Gamma^2 L'$

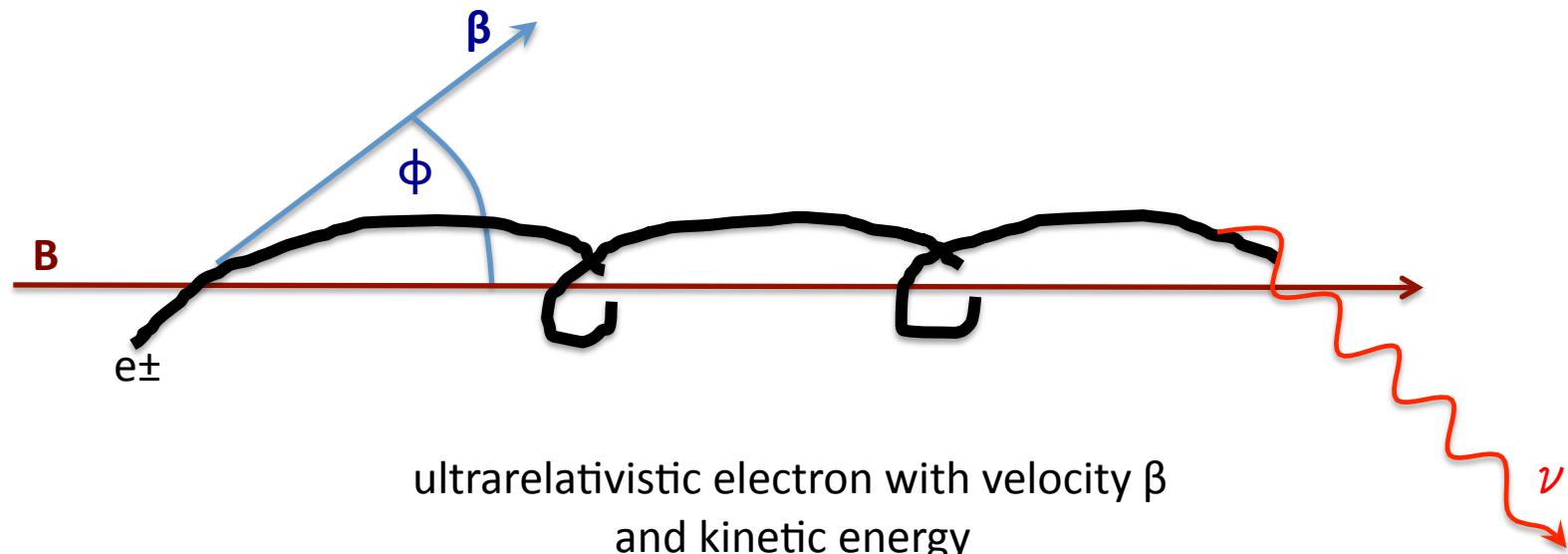


for  $\delta = 1$  and  $L'_{\nu'} \propto \nu'^{-\alpha}$

$$S_\nu = L_\nu / 4\pi d_L^2 = (1+z) L'_{\nu'} / 4\pi d_L^2 = (1+z)^{1-\alpha} L'_\nu / 4\pi d_L^2$$

$$[\nu S_\nu] = [\nu L_\nu] / 4\pi d_L^2 = [\nu' L'_{\nu'}] / 4\pi d_L^2 = (1+z)^{1-\alpha} [\nu L'_\nu] / 4\pi d_L^2$$

## II. Synchrotron Emission



ultrarelativistic electron with velocity  $\beta$   
and kinetic energy

$$E_e = \gamma m_e c^2 \gg m_e c^2$$

gyrating along the magnetic field  $B$   
with pitch angle  $\phi$

## II. Single Electron

gyrofrequency  $\omega_B$

$$\omega_B = \frac{eB}{\gamma m_e c} \quad (31)$$

radius of gyration projected on a plane  
normal to  $B$  and the Larmor radius  $r_L$

$$r_B = \frac{c}{\omega_B} \sin \phi \equiv r_L \sin \phi \quad (32)$$

total synchrotron power  
radiated by a single electron

$$P_{\text{syn}}(\phi) = \frac{1}{4\pi} \sigma_T c \gamma^2 B^2 \sin^2 \phi \quad (33)$$

$$P_{\text{syn}} \equiv \frac{1}{2} \int_0^\pi P_{\text{syn}}(\phi) \sin \phi d\phi = \frac{1}{6\pi} \sigma_T c \gamma^2 B^2 \quad (34)$$

lifetime of the synchrotron-radiating  
electron (synchrotron cooling timescale)

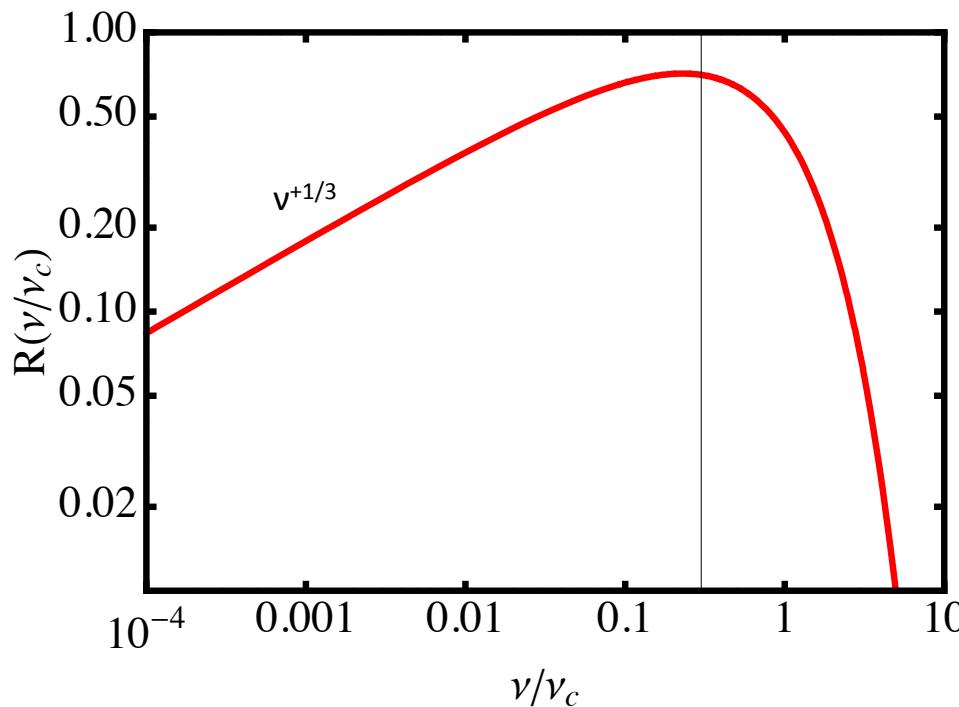
$$\tau_{\text{syn}} \equiv \frac{\gamma m_e c^2}{P_{\text{syn}}} = \frac{6\pi m_e c}{\sigma_T \gamma B^2} \quad (35)$$

## II. Spectral Distribution

$$P_{\text{syn}}(\nu) = \frac{\sqrt{3} e^3 B}{m_e c^2} \times \mathcal{R}\left(\frac{\nu}{\nu_c}\right) \quad (36)$$

$$\nu_c \equiv \frac{3 e B}{4 \pi m_e c} \gamma^2 \quad (37)$$

$$\mathcal{R}(x) \equiv \frac{x^2}{2} K_{4/3}\left(\frac{x}{2}\right) K_{1/3}\left(\frac{x}{2}\right) - 0.3 \frac{x^3}{2} \left[ K_{4/3}^2\left(\frac{x}{2}\right) - K_{1/3}^2\left(\frac{x}{2}\right) \right] \quad (38)$$



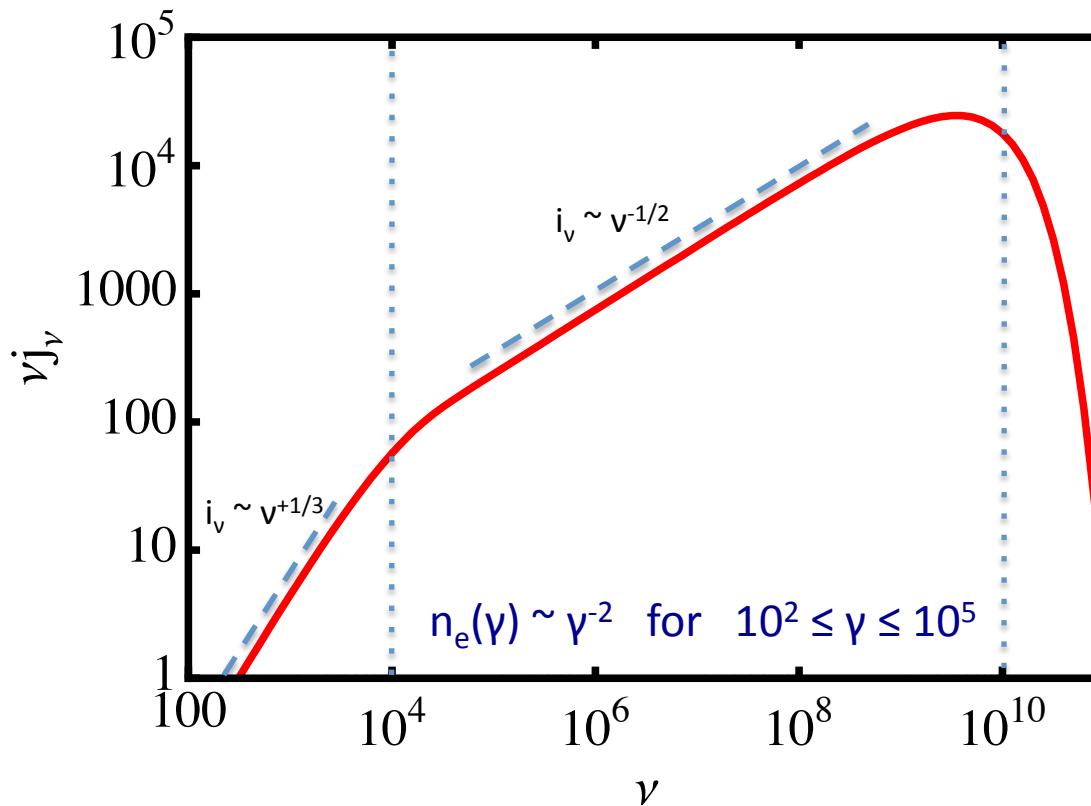
characteristic synchrotron frequency  
 $\nu_c \approx 4 (B/\mu\text{G}) \gamma^2 [\text{Hz}]$

$R(\nu) \sim \nu^{+1/3}$  for  $\nu < \nu_c$   
but sometimes we approximate  
 $R(\nu) \sim \delta(\nu - \nu_c)$

## II. Ensemble of Electrons

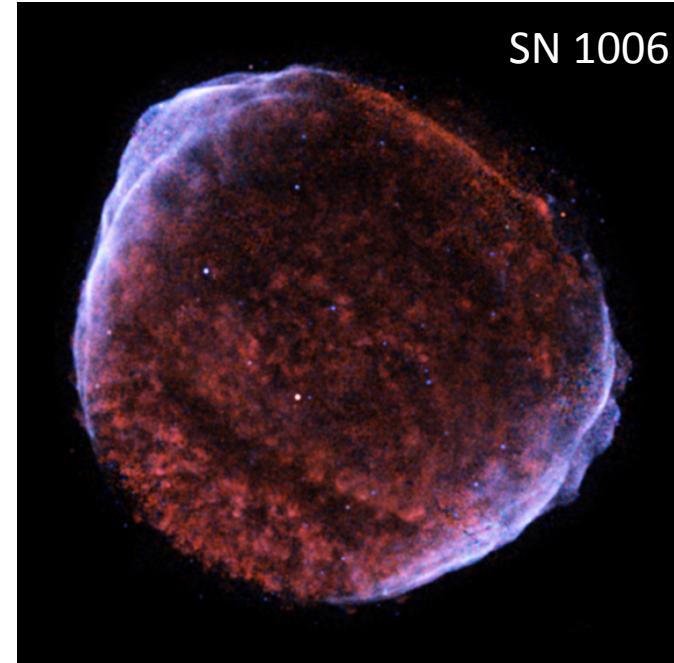
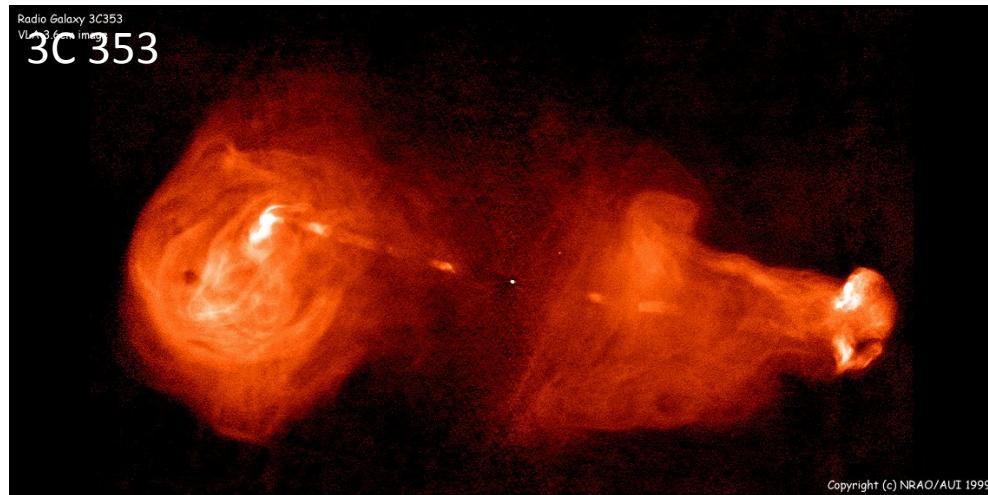
$$j_\nu = \frac{1}{4\pi} \int P_{\text{syn}}(\nu) n_e(\gamma) d\gamma \quad (39)$$

if  $n_e(\gamma) \propto \gamma^{-p}$  for  $\gamma_{\min} \leq \gamma \leq \gamma_{\max}$  (40)  
then  $j_\nu \propto \nu^{-(p-1)/2}$  for  $\nu_c \gamma_{\min}^2 \ll \nu \ll \nu_c \gamma_{\max}^2$



## II. Summary

- Synchrotron cooling timescale  $\tau_{\text{syn}} \sim 1 / \gamma B^2$
- Emitted synchrotron power  $P_{\text{syn}} \sim \gamma^2 B^2$
- Characteristic synchrotron frequency  $\nu \approx 4 (B/\mu\text{G}) \gamma^2 [\text{Hz}]$
- Synchrotron spectral index ( $S_\nu \sim \nu^{-\alpha}$ )  $\alpha = (p-1)/2 > -1/3$
- For a power-law electron energy distribution, synchrotron continuum is a power-law within only limited frequency range; electron breaks and cut-offs result in smoothly curved synchrotron spectra!



Radio synchrotron emission of AGN jets and lobes  
X-ray synchrotron emission of SNRs

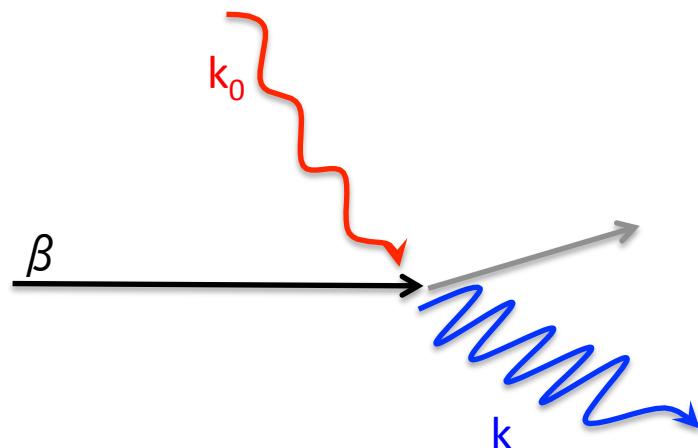
### III. Inverse-Compton Emission

ultrarelativistic electron with velocity  $\vec{\beta}$  and energy  $E_e = \gamma m_e c^2 \gg m_e c^2$   
incident (target) photon with wave vector  $\vec{k}_0$  and energy  $\varepsilon_0 \equiv \epsilon_0 m_e c^2$   
scattered photon with wave vector  $\vec{k}$  and energy  $\varepsilon \equiv \epsilon m_e c^2$

$\psi_0 \equiv \cos^{-1} \eta_0$  — angle between  $\vec{k}_0$  and  $\vec{\beta}$

$\psi \equiv \cos^{-1} \eta$  — angle between  $\vec{k}$  and  $\vec{\beta}$

$\chi \equiv \cos^{-1} \kappa$  — angle between  $\vec{k}_0$  and  $\vec{k}$



### III. Kinematics

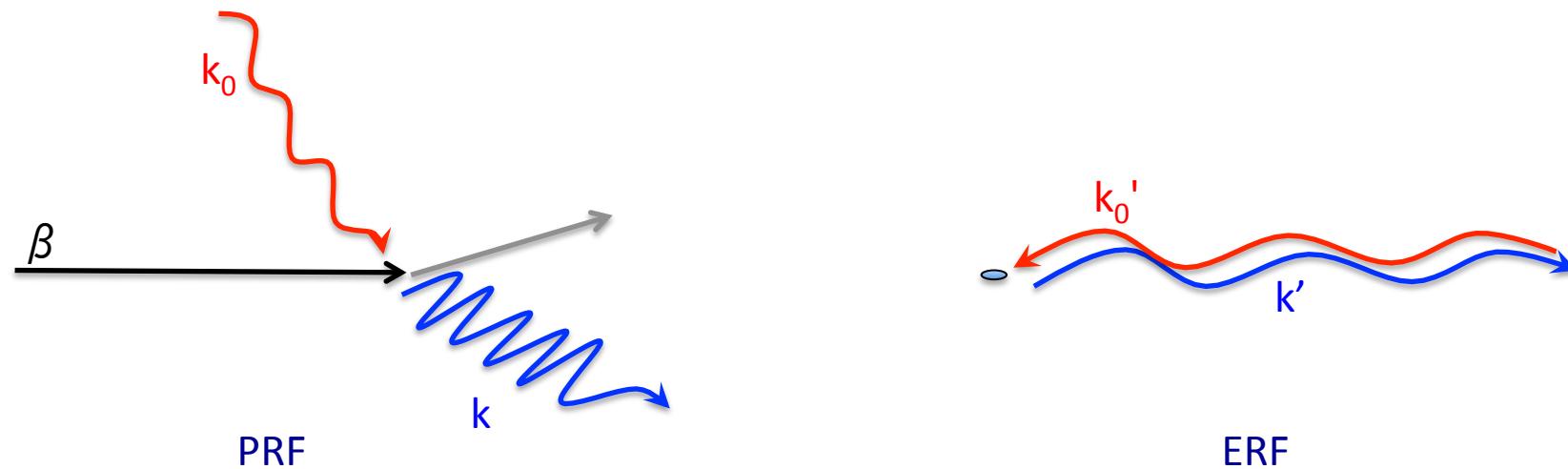
transformation to the ERF  $\epsilon'_0 = \gamma \epsilon_0 (1 - \beta \eta_0)$  (41)

scattering (4-momentum conservation)  $\epsilon' = \frac{\epsilon'_0}{1 + \epsilon'_0 (1 - \kappa')}$  (42)

transformation back to the PRF  $\epsilon = \gamma \epsilon' (1 + \beta \eta')$  (43)

for  $\epsilon'_0 < 1$  one has  $\epsilon' \approx \epsilon'_0$  (elastic scattering)

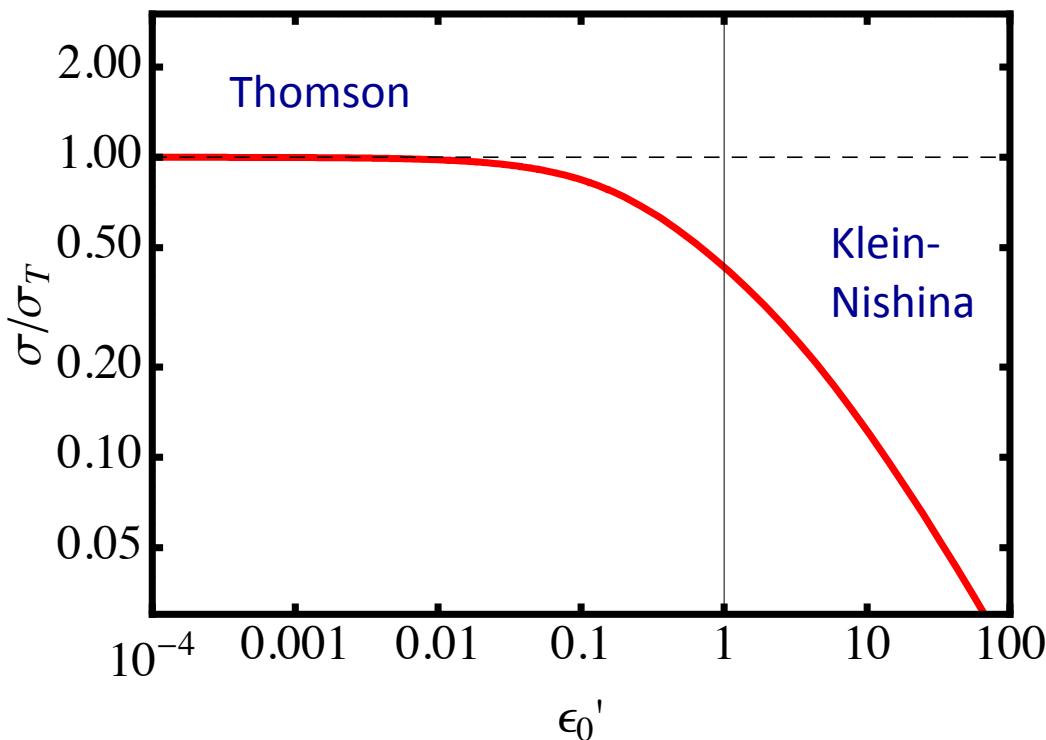
for  $\beta \approx 1$  one has  $\eta'_0 \approx -1$ ,  $\eta' = -\kappa'$  and  $\eta \approx 1$  ('head-on approximation')



### III. Thomson and KN Regimes

$$\frac{d^2\sigma}{d\Omega' d\epsilon'} = \frac{3\sigma_T}{16\pi} \left( \frac{\epsilon'}{\epsilon'_0} \right)^2 \left( \frac{\epsilon'_0}{\epsilon'} + \frac{\epsilon'}{\epsilon'_0} - (1 - \kappa'^2) \right) \times \delta \left[ \epsilon' - \frac{\epsilon'_0}{1 + \epsilon'_0 (1 - \kappa')} \right] \quad (44)$$

$$\begin{aligned} \sigma &\equiv \oint d\Omega' \int d\epsilon' \frac{d^2\sigma}{d\Omega' d\epsilon'} = \\ &= \frac{3\sigma_T}{8\epsilon'_0} \left[ \left( 1 - \frac{2}{\epsilon'_0} - \frac{2}{\epsilon'^2_0} \right) \ln(1 + 2\epsilon'_0) + \frac{1}{2} + \frac{4}{\epsilon'_0} - \frac{1}{2(1 + 2\epsilon'_0)^2} \right] \approx \\ &\approx \begin{cases} \frac{\sigma_T}{8\epsilon'_0} \ln(2e^{1/2}\epsilon'_0) & \text{for } \epsilon'_0 \ll 1 \\ \frac{3\sigma_T}{8\epsilon'_0} \ln(2e^{1/2}\epsilon'_0) & \text{for } \epsilon'_0 \gg 1 \end{cases} \end{aligned} \quad (45)$$



Klein-Nishina suppression is often modeled as a sharp cut-off

### III. Isotropic Electron Distribution

$$P_{\text{ic}}(\nu) = c h \epsilon \int d\epsilon_0 \oint d\Omega_0 \oint d\Omega_e (1 - \beta \eta_0) n(\epsilon_0, \Omega_0) \frac{d^2\sigma}{d\Omega d\epsilon} \quad (46)$$

$$\tau_{\text{ic}} \equiv \frac{\gamma m_e c^2}{\int d\nu P_{\text{ic}}(\nu)} \quad (47)$$

$$j_\nu = \frac{1}{4\pi} \int P_{\text{ic}}(\nu) n_e(\gamma) d\gamma \quad (48)$$

for  $n(\epsilon_0, \Omega_0) = n(\epsilon_0)/4\pi$

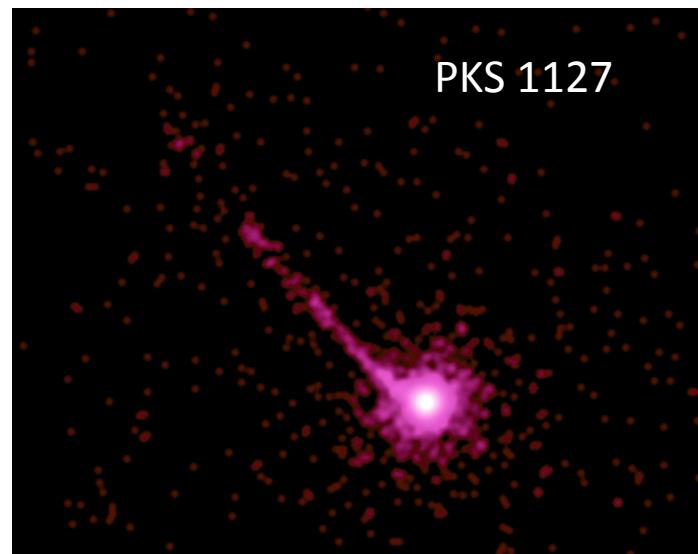
$$j_\nu = \frac{3 c h^2 \sigma_T \nu}{16\pi m_e c^2} \int d\epsilon_0 \int d\gamma n(\epsilon_0) n_e(\gamma) \frac{f(\epsilon, \epsilon_0, \gamma)}{\gamma^2 \epsilon_0} \quad (49)$$

$$f(\epsilon, \epsilon_0, \gamma) = 2q \ln q + q + 1 - 2q^2 + \frac{(Qq)^2 (1-q)}{2(1+Qq)} \quad (50)$$

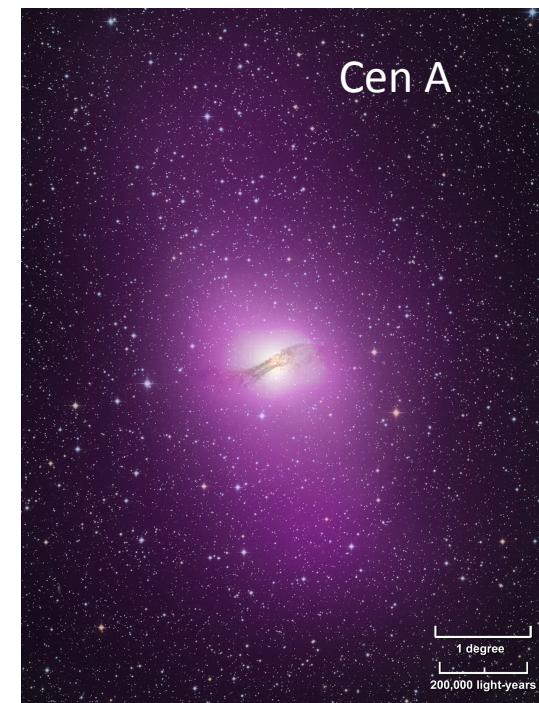
where  $Q \equiv 4\epsilon_0\gamma$ ,  $q \equiv \epsilon/4\epsilon_0\gamma(\gamma - \epsilon)$  and  $1/4\gamma^2 \leq q \leq 1$

### III. Summary

- IC/TR cooling timescale  $\tau_{\text{ic}} \sim 1 / \gamma U_0$
- Emitted IC/TR power  $P_{\text{ic}} \sim \gamma^2 U_0$
- Characteristic IC/TR energy  $\varepsilon \approx \varepsilon_0 \gamma^2$
- IC/TR spectral index ( $S_\nu \sim \nu^{-\alpha}$ )  $\alpha = (p-1)/2 > -1$
- For a power-law electron energy distribution, IC continuum is a power-law within only limited frequency range; electron breaks and cut-offs, as well as KN effects, result in smoothly curved IC spectra!



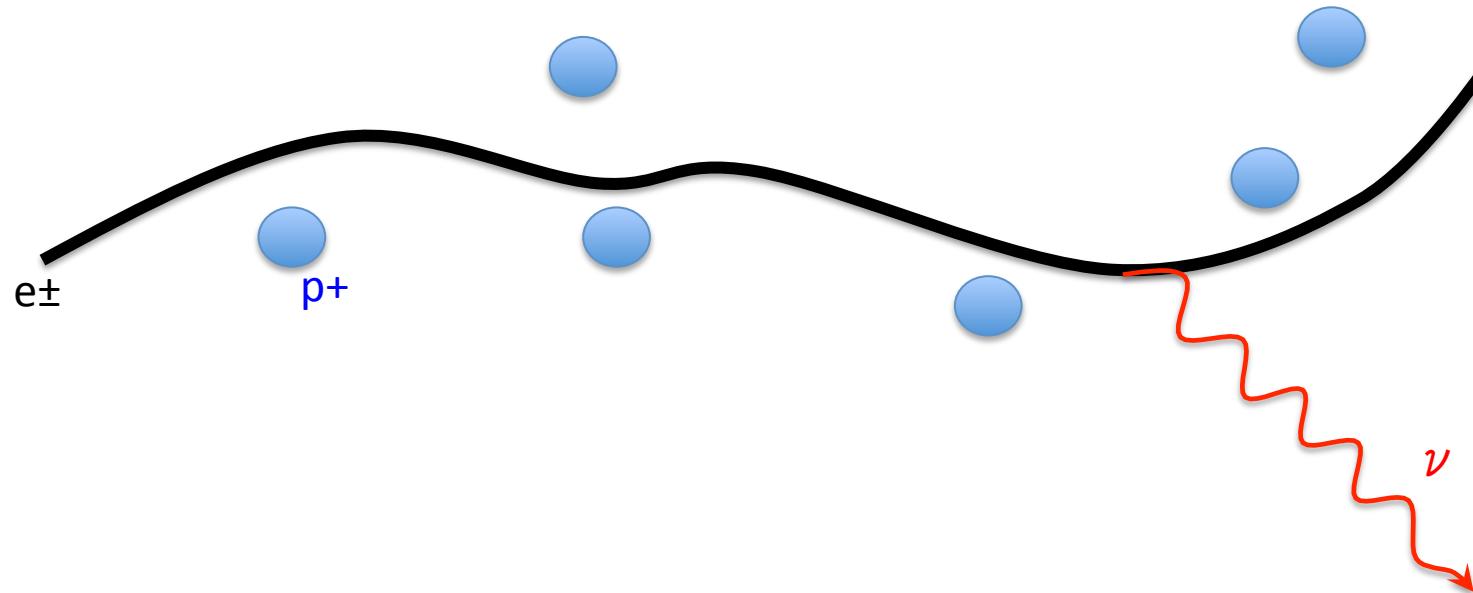
X-ray IC emission of AGN jets  
γ-ray IC emission of AGN lobes



# IV. Thermal Bremsstrahlung

Electron-ion bremsstrahlung (“free-free emission”):  
radiation of the thermal electrons accelerated in the Coulomb field of thermal ions

$$n_{\text{th}}(v) dv = 4\pi n_g v^2 \left(\frac{m_e}{2\pi kT}\right)^{3/2} e^{m_e v^2 / 2kT} dv \quad (51)$$

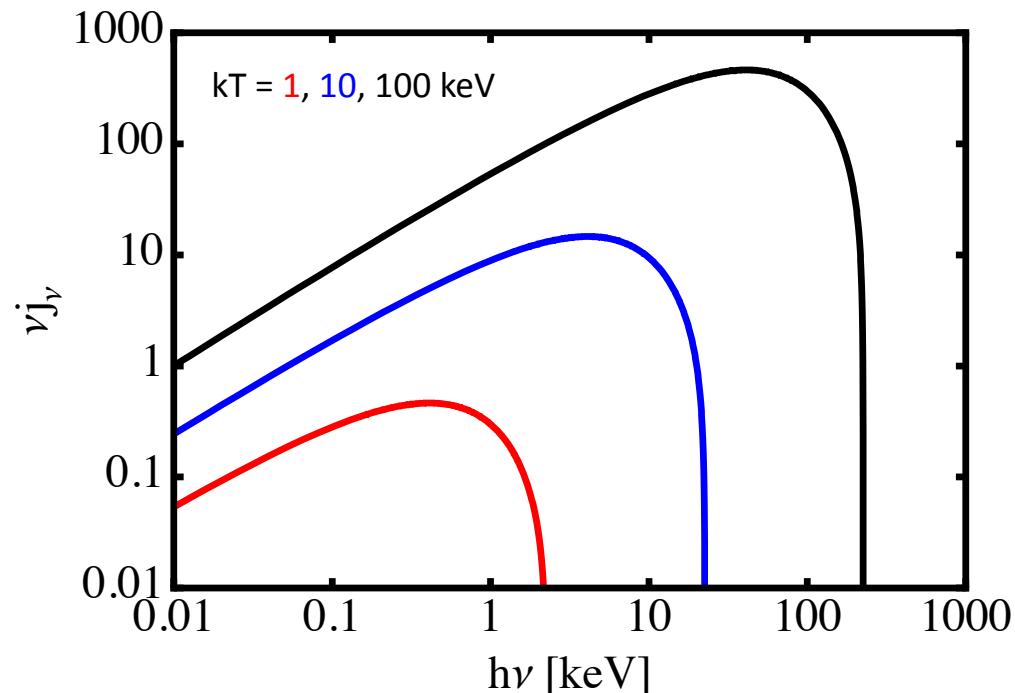


## IV. Free-Free Emissivity

$$P_{\text{ff}}(\nu) = \frac{32 e^6}{\sqrt{3} m_e c^3} \left( \frac{2\pi}{3k m_e} \right)^{1/2} n_g^2 T^{-1/2} e^{-h\nu/kT} \ln \left[ 2.25 \frac{kT}{h\nu} \right] \quad (52)$$

$$j_\nu = \frac{1}{4\pi} P_{\text{ff}}(\nu) \quad (53)$$

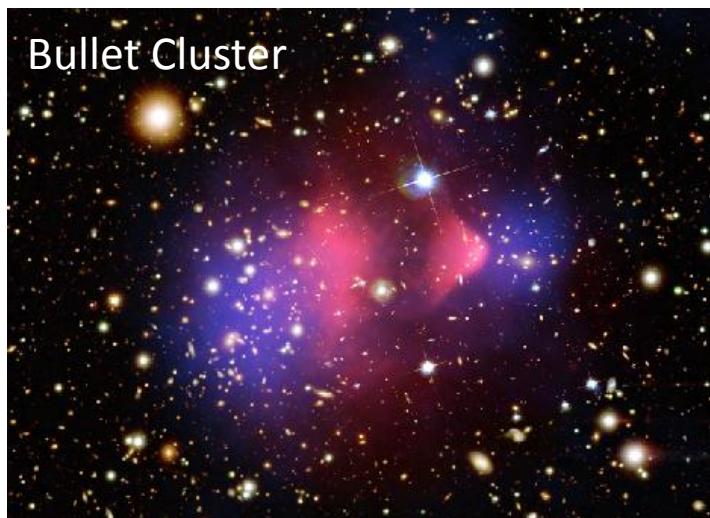
Detection of the thermal bremsstrahlung emission enables to diagnose thermal plasma (density and temperature) in astrophysical sources of high-energy radiation.



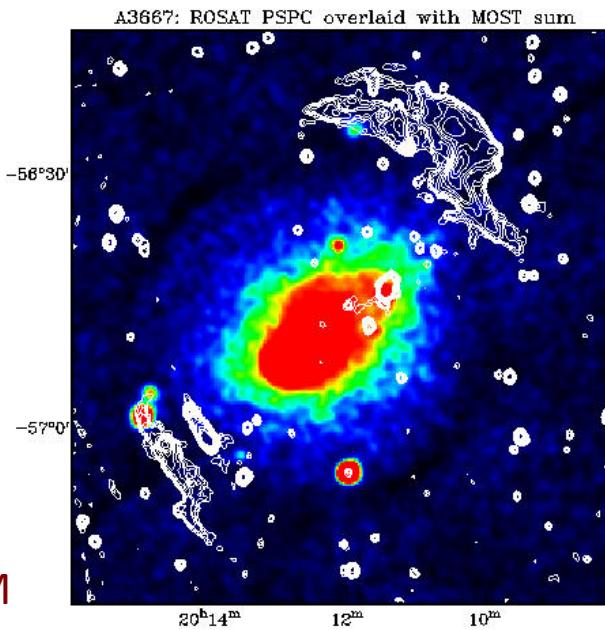
# IV. Summary

- Bremsstrahlung cooling timescale
- Emitted bremsstrahlung power
- Critical (maximum) photon energy
- Bremsstrahlung spectral index ( $S_v \sim v^{-\alpha}$ )
- Thermal and non-thermal bremsstrahlung may compete with the IC process within the MeV/GeV photon energy range in the case of the systems characterized by high density of thermal gas (starbursts, SNRs)

$$\begin{aligned}\tau_{ic} &\sim 1 / n_g \\ P_{ic} &\sim T^{1/2} n_g^2 \\ \varepsilon_{cr} &\approx kT \\ \alpha &\approx 0\end{aligned}$$

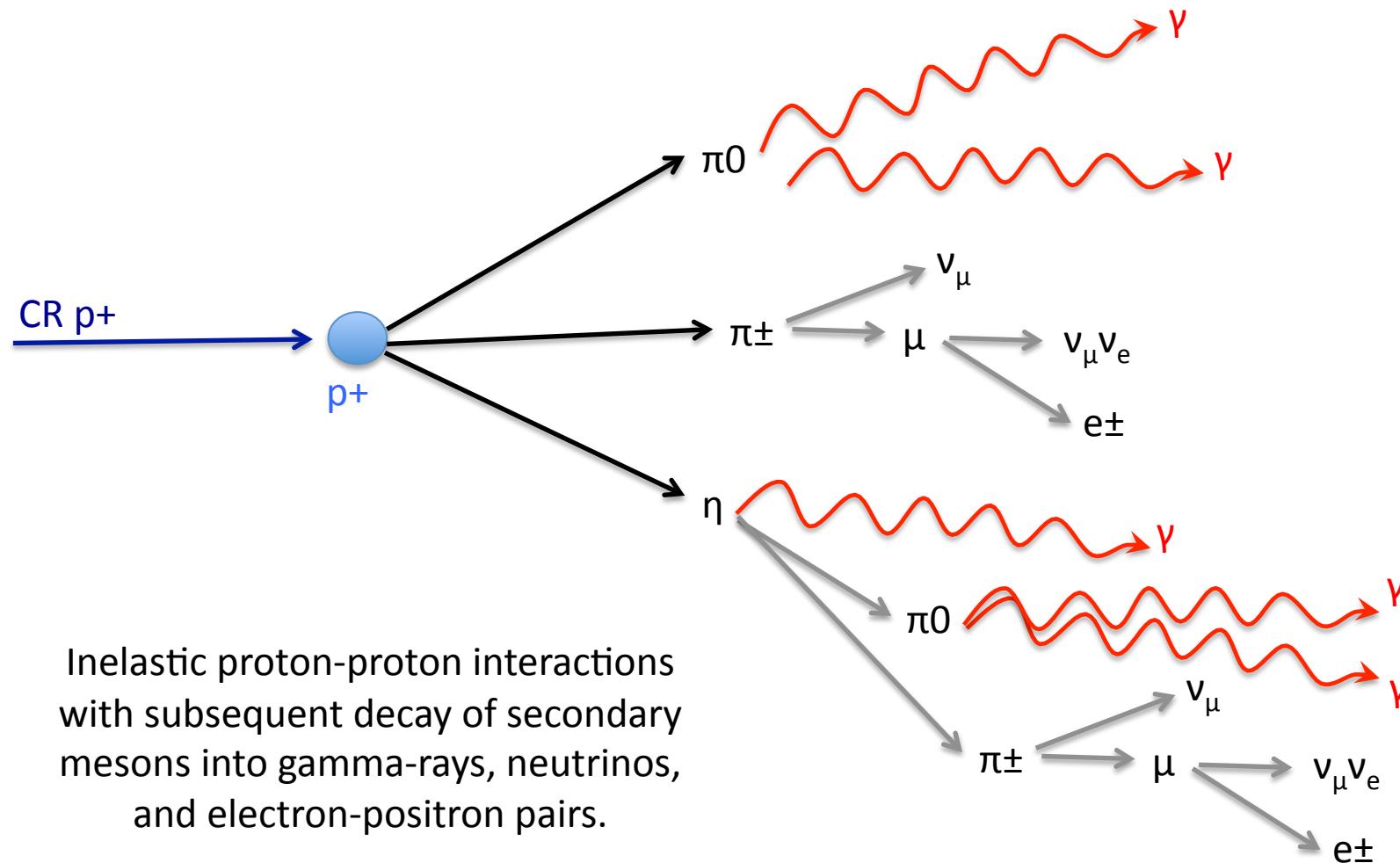


X-ray bremsstrahlung emission of ICM



Right Ascension (J2000)

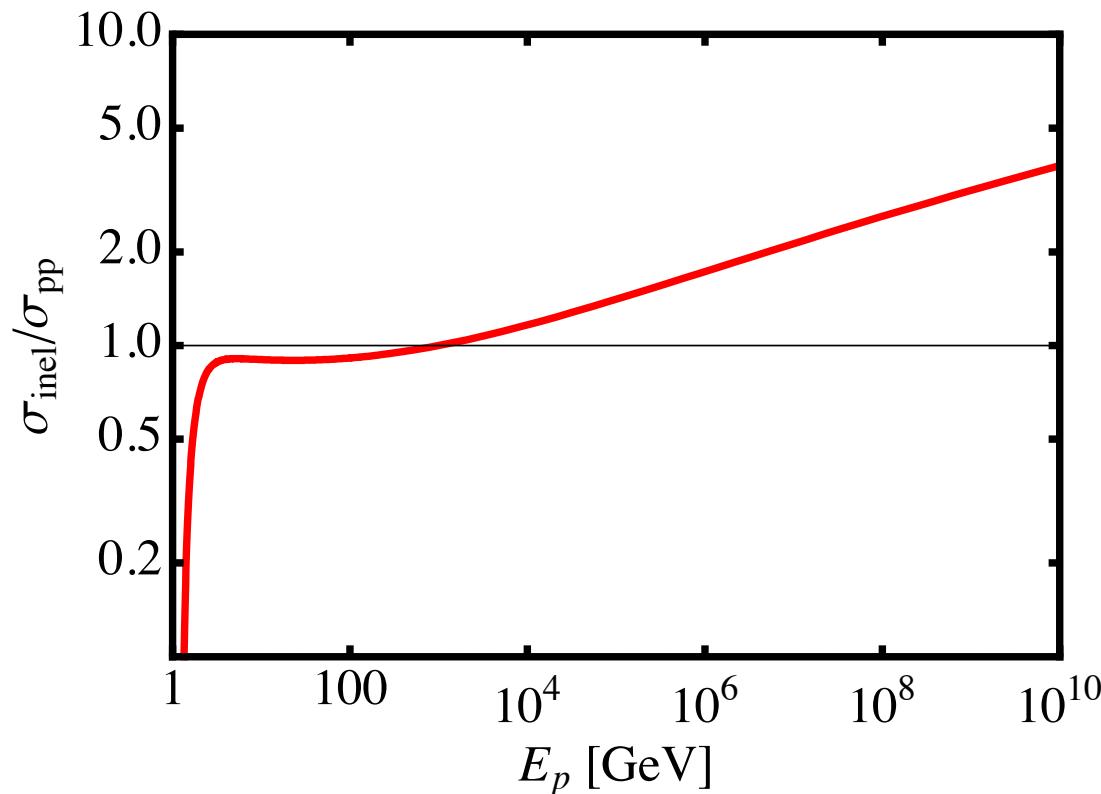
# V. Proton-Proton Interactions



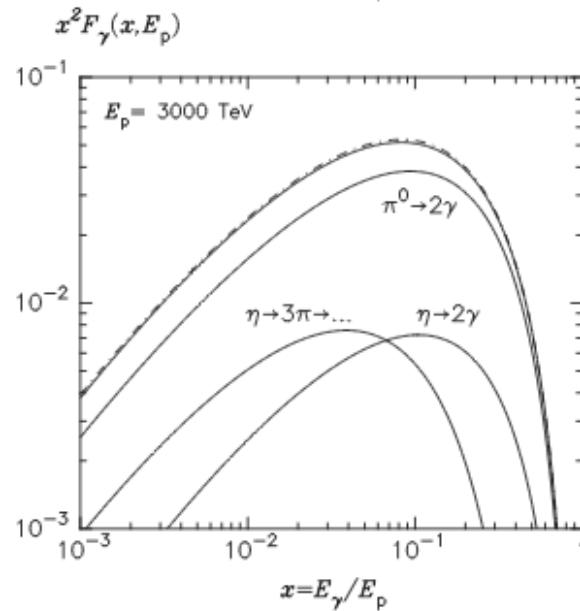
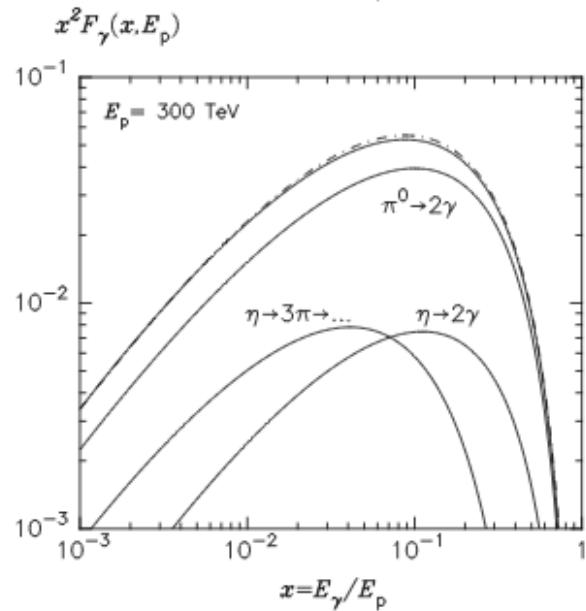
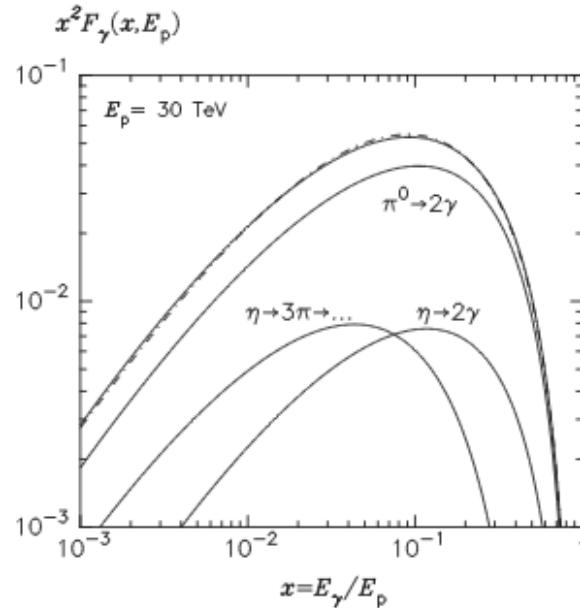
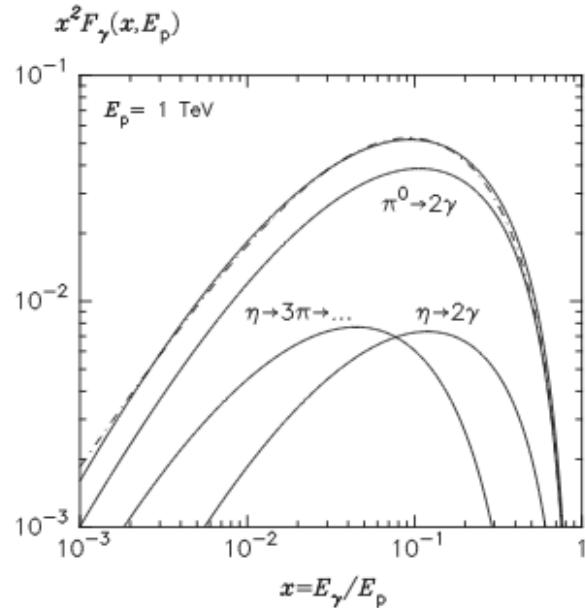
## V. Inelastic Cross Section

$$\frac{\sigma_{\text{inel}}(E_p)}{\sigma_{pp}} \simeq \left[ 1 + 0.055 \ln\left(\frac{E_p}{\text{TeV}}\right) + 0.007 \ln^2\left(\frac{E_p}{\text{TeV}}\right) \right] \times \left[ 1 - \left(\frac{E_{\text{th}}}{E_p}\right)^4 \right]^2 \quad (54)$$

$$\sigma_{pp} = 34.3 \text{ mb} \quad \text{and} \quad E_{\text{th}} = m_p c^2 + 2m_\pi c^2 + m_\pi^2 c^2 / 2m_p \simeq 1.22 \text{ GeV} \quad (55)$$



# V. Gamma-Ray Production

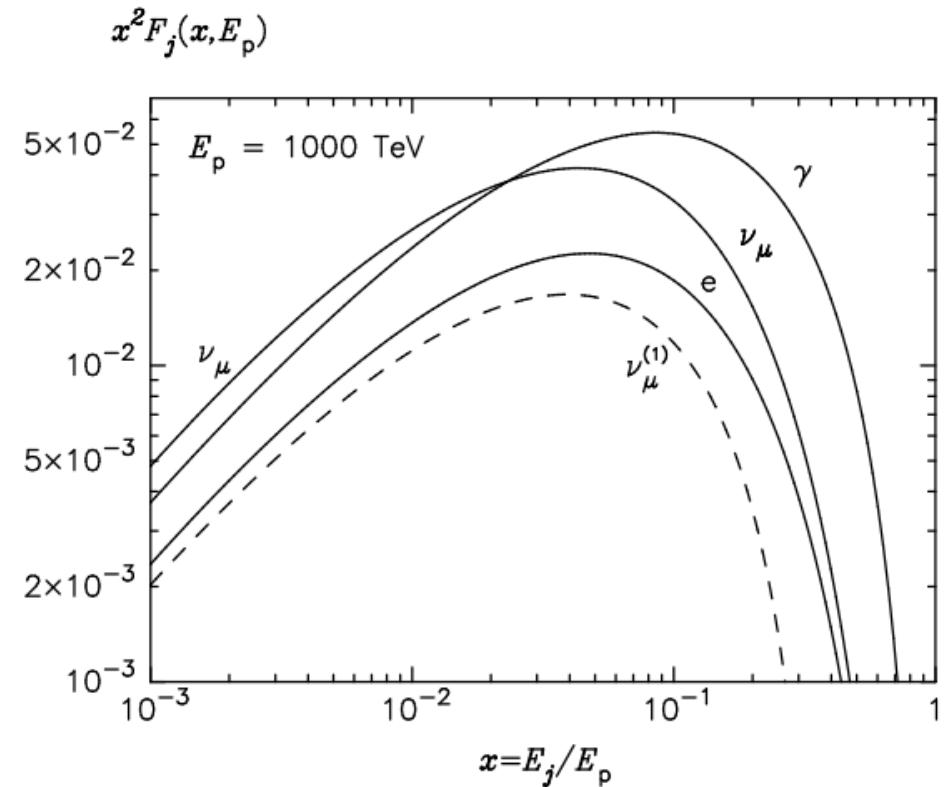
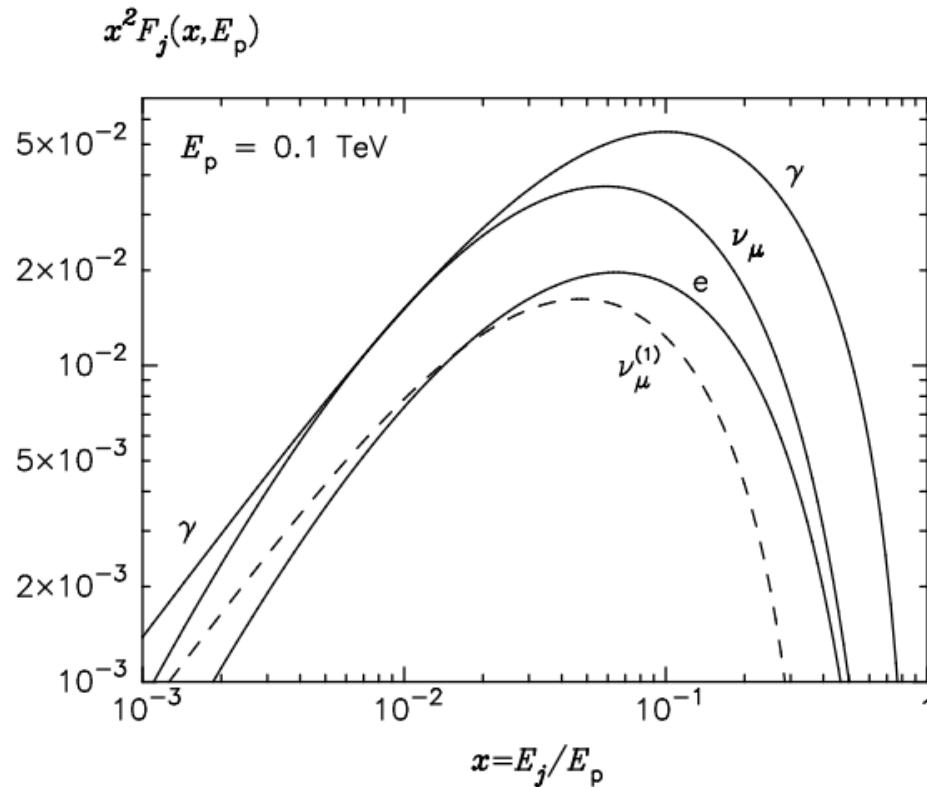


~6 gamma-rays with energies

$$\varepsilon_\gamma \approx 0.1 E_p$$

per  $pp$  collision are produced  
(plots from Kelner et al. 2006)

# V. Secondary Particles



(from Kelner et al. 2006)

# V. Broad-Band Spectra

“delta-approximation”

$$\dot{n}_\gamma(\varepsilon_\gamma) = 2 \int_{E_{\min}} \frac{q_\pi(E_\pi)}{\sqrt{E_\pi^2 - m_\pi^2 c^4}} dE_\pi \quad (56)$$

$$q_\pi(E_\pi) \simeq 5.9 c n_g \times J_p(m_p c^2 + 5.9 E_\pi) \times \sigma_{\text{inel}}(m_p c^2 + 5.9 E_\pi) \quad (57)$$

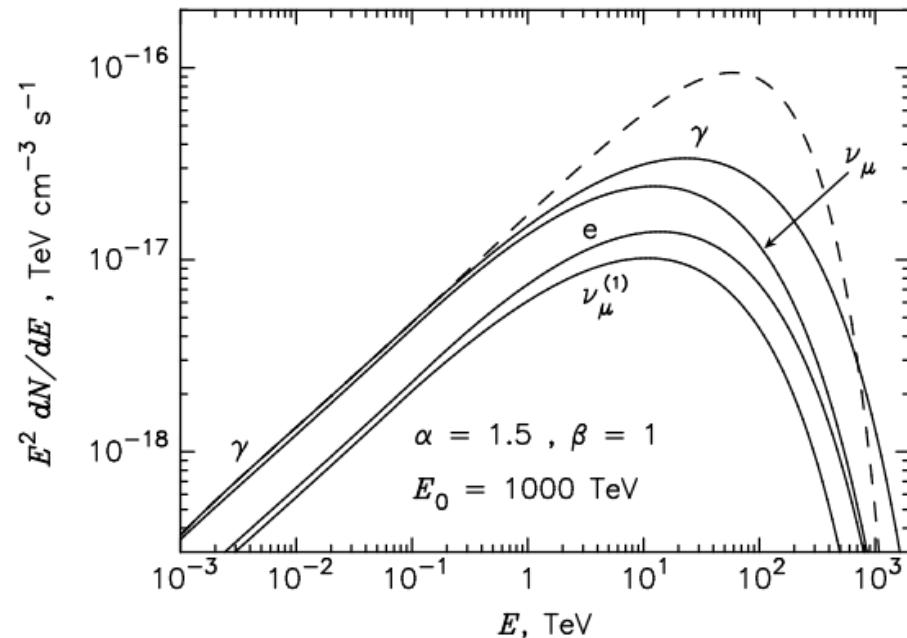
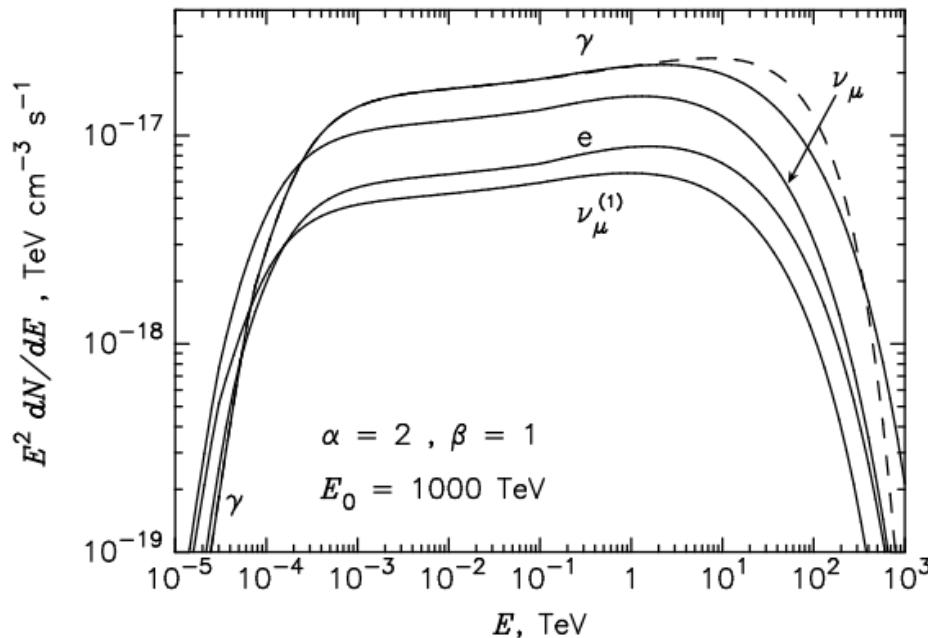
$$E_{\min} = \varepsilon_\gamma + \frac{m_\pi^2 c^4}{4\varepsilon_\gamma} \quad \text{and} \quad m_\pi c^2 = 135 \text{ MeV} \quad (58)$$

$$j_\varepsilon = \frac{\varepsilon_\gamma}{4\pi} \dot{n}_\gamma(\varepsilon_\gamma) \quad (59)$$

Energy spectra of secondary particles closely follow energy spectrum of CR protons

$$J_p(E_p) \sim E_p^{-\alpha} \exp[-(E_p/E_0)^\beta]$$

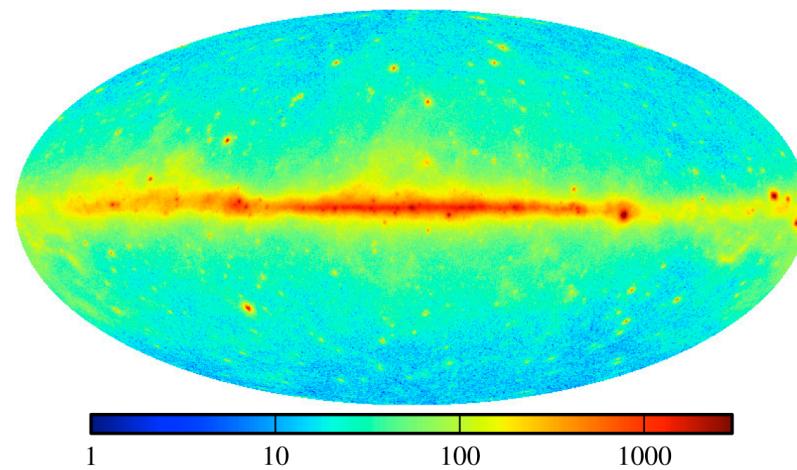
(from Kelner et al. 2006)



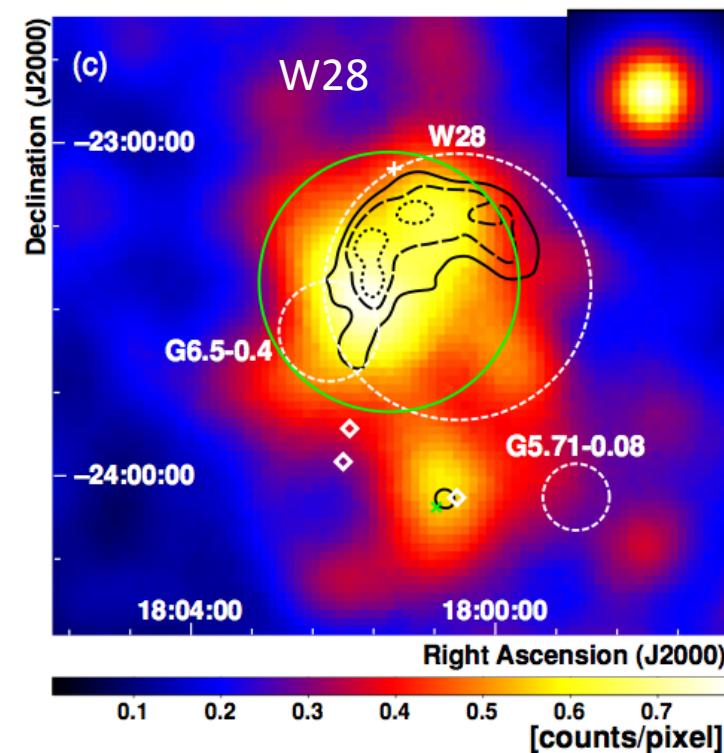
# V. Summary

- PP interaction timescale
- Energies of produced  $\gamma$ -rays
- $\gamma$ -ray photon index ( $dN_\gamma/d\epsilon_\gamma \sim \epsilon^{-\Gamma}$ )
- Low-energy cutoff in  $\gamma$ -ray spectra around 130 MeV expected!
- Production of  $\gamma$ -rays always accompanied by the production of neutrinos and secondary  $e^\pm$  pairs.

$$\begin{aligned}\tau_{pp} &\sim 1 / n_g \\ \epsilon_\gamma &\approx 0.1 E_p \\ \Gamma_\gamma &= \alpha_\gamma + 1 \approx \alpha_p\end{aligned}$$



Diffuse GeV emission of the Galaxy  
GeV emission of middle-age SNRs



# VI. Photo-Meson Production

$$\tau_{pp} \simeq [c \times \sigma_{pp} \times n_g]^{-1}$$

$$\sigma_{pp} = 34.3 \text{ mb}$$

$$\tau_{p\gamma} \simeq [c \times \langle \sigma_{p\gamma} K_{p\gamma} \rangle \times n_0^*]^{-1}$$

$$\langle \sigma_{p\gamma} K_{p\gamma} \rangle \simeq 0.07 \text{ mb}$$

$$n_0^* = \int_{\varepsilon_{th}} d\varepsilon_0 n_0(\varepsilon_0)$$

$$\varepsilon_{th}(E_p) = \frac{m_\pi c^2}{E_p/m_p c^2}$$

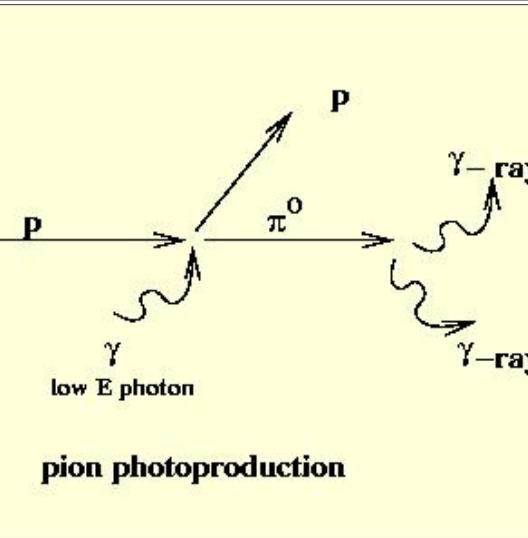
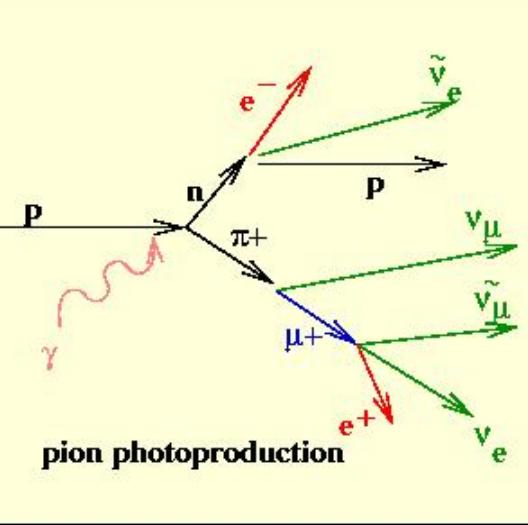
number density of target photons

product of photo-meson cross section and inelasticity parameter averaged over the resonant energy range

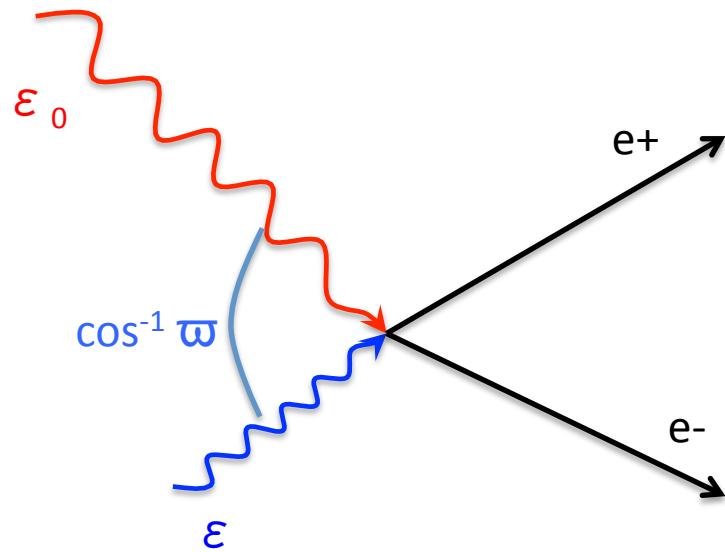
(60)  
(61)

(62)  
(63)

(64)  
(65)



# VII. Photon-Photon Annihilation



$$\varepsilon \times \varepsilon_0 > (m_e c^2)^2$$

# VII. Energy “Resonance”

$$\sigma_{\gamma\gamma} = \frac{3\sigma_T}{16} (1 - b^2) \left[ (3 - b^4) \ln \left( \frac{1+b}{1-b} \right) - 2b(2 - b^2) \right] \quad (66)$$

$$b \equiv \left( 1 - \frac{2}{\epsilon \epsilon_0 (1 - \varpi)} \right)^{1/2} \quad (67)$$

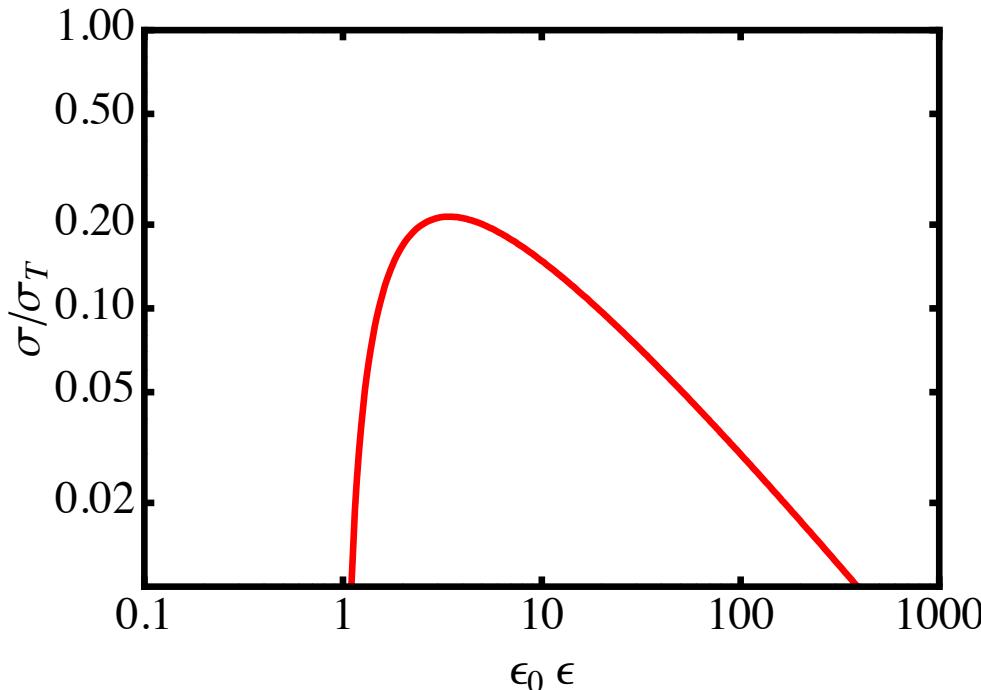
velocity of the created  $e\pm$  in the center-of-momentum frame

$$\begin{aligned} \langle \sigma_{\gamma\gamma} \rangle &\equiv \frac{1}{2} \int_{-1}^{1-(2/\epsilon\epsilon_0)} d\varpi (1 - \varpi) \sigma_{pp} \approx \\ &\approx 0.65 \sigma_T \frac{(\epsilon\epsilon_0)^2 - 1}{(\epsilon\epsilon_0)^3} \ln(\epsilon\epsilon_0) H[\epsilon\epsilon_0 - 1] \sim \frac{2}{3} \sigma_T \delta(\epsilon\epsilon_0 - 2) \end{aligned} \quad (68)$$

Optical depth:

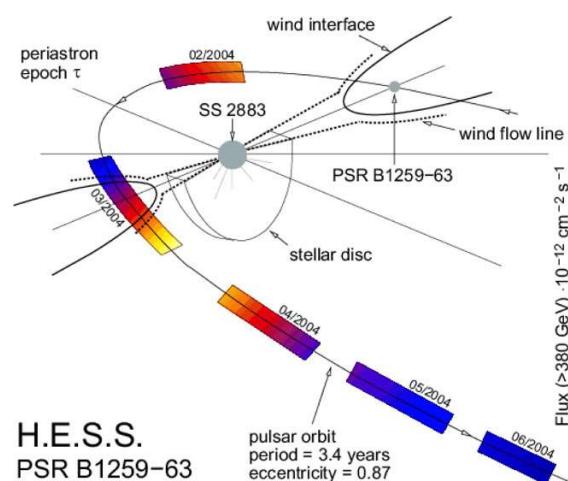
$$\tau_{\gamma\gamma} = \int_0^L d\ell \int_{1/\epsilon_0}^{\infty} d\epsilon_0 n_0(\epsilon_0) \langle \sigma_{\gamma\gamma} \rangle \quad (69)$$

$$\rightarrow \tau_{\gamma\gamma} (\epsilon = 2/\epsilon_0) \sim \frac{1}{3} \sigma_T L n_0 \quad (70)$$

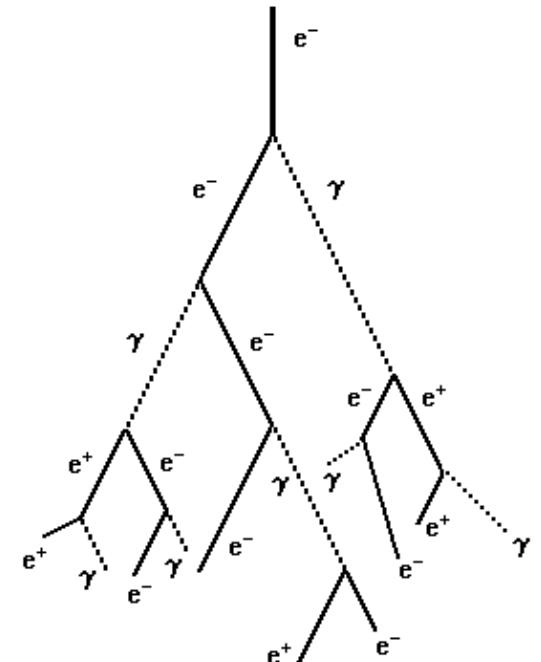
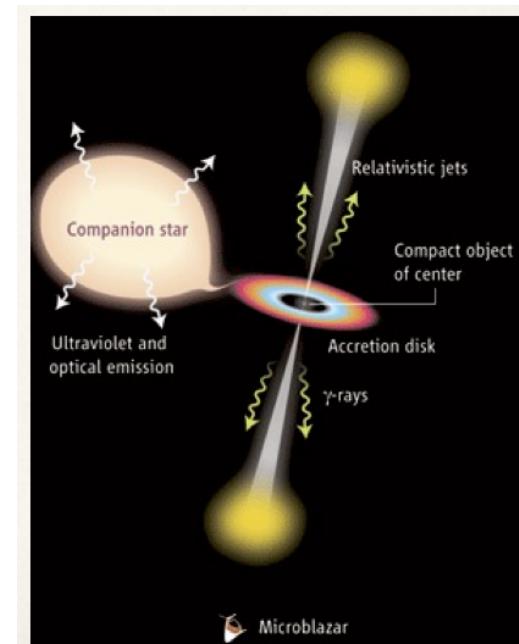


# VII. Summary

- Energy “resonance”  $\epsilon_0 \epsilon \sim 2 (m_e c^2)^2$ , so that the 0.1 GeV – 10 TeV  $\gamma$ -rays are absorbed most efficiently by X-ray – infrared photons
- Generation of secondary ultrarelativistic pairs  $E_{e\pm} \approx \epsilon/2 \gg m_e c^2$
- The absorbed power is not lost, but reprocessed to lower-frequencies (via synchrotron and IC cooling of secondary pairs).
- Development of linear and isotropic cascades.
- Absorption may lead to formation of breaks and cut-offs in  $\gamma$ -ray spectra of astrophysical sources.



GeV/TeV emission of  
Gamma-ray Binaries



Credit: Mirabel Science, 2012, 335, 175